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Long-Range Performance of Suboptimal Periodic Hypersonic Cruise Trajectories

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Introduction

THE discovery of periodic cruise trajectories for hypersonic flight has evolved from extensive analytical and computational optimization studies performed to determine possible trajectory types that could achieve better fuel consumption savings. With this goal in mind, several researchers have found suboptimal and optimal periodic cruise trajectories that achieve better fuel consumption savings between two destinations over steady-state cruise for a single period. Results from these past studies suggest that fuel consumption savings of 8 to 45% (Refs. 1–6) are possible over a single period. The large difference comes from the use of damped periodic trajectories as compared to purely periodic trajectories.[5] However, to achieve long-range flight, multiple periods will be required. Thus, the question that arises is how does the large performance benefit of damped periodic hypersonic cruise trajectories over a single period vary for multiple periods, as compared to steady-state cruise and purely periodic cruise. The purpose of this Note is to examine this performance by optimizing each period of the trajectory after applying various types of boundary conditions.

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Table 1 Assumptions for performance comparisons

Parameterized altitudes	Comments
	Method 1
Eq. (1)	AOA continuity
Eq. (2)	Initial and final energy ratios are equivalent
	Method 2
Eq. (1)	AOA continuity
Eq. (3)	Initial and final energy ratios are equivalent
	Method 3
Same as method 1	AOA continuity, initial energy ratio equal to optimal steady-state energy ratio

Parameterized Altitude Profiles

Because periodic hypersonic flight trajectories appear to exhibit harmonic behavior, a parameterized form of the altitude profile consisting of only a few harmonics can come quite close to capturing the salient features of the optimal trajectory. Because these harmonics are defined by constants, a suboptimal solution is possible by simply determining the parameters of the altitude profile. This results in less computational effort than a complete optimization of the minimum fuel-consumption rate two-point boundary value problem (TPBVP). The details of this optimization can be found in work done by Chuang and Morimoto[4] and von Eggers Rudd, et al.[5] For multiple periods, an optimization over the complete range of periods would be desirable, but would be complicated and computationally intensive. In the present work, the periods are optimized one at a time and the solutions pieced together. Five methods are evaluated for a variety of parameterized altitude profiles. Each of these methods are described hereafter and summarized in Table 1.

Method 1

The first method for a long range enforces the states to be continuous at each period's boundary. This produces a realistic flight profile. For steady-state cruise, only Mach number and initial altitude need to be optimized (this steady-state optimization holds for all methods). Periodic hypersonic cruise uses the altitude parameterization given by

$$h = h_a \cos(2\pi / r_f)r + h_b \cos(4\pi / r_f)r + h_c \tag{1}$$

For the first period, the variables to be optimized include the harmonic constants h_a, h_b, and h_c; the initial Mach number M_0; throttle on r_u; throttle off r_d; and final range r_f to minimize the cost function given in Eq. (4). For each period thereafter, only r_u, r_d, and r_f are optimized so that the initial and final states are always equivalent.

The damped periodic optimization uses an altitude profile as a function of range r given by[5]

$$h = \exp[-\eta(r/r_f)][h_a \cos(2\pi/r_f)r + h_b \cos(4\pi/r_f)r] + h_c \tag{2}$$

Variables to be optimized for the first period include h_a, h_b, h_c, M_o, r_u, r_d, r_f, and the damping term η. Beyond the first period, only r_u and r_d are optimized. The variable r_f must stay constant over multiple periods if continuity of angle of attack (AOA) is desired.

Method 2

As damped periodic trajectories propagate over long ranges, they will eventually damp out to a mean steady-state cruise altitude. The altitude parameterization given by Eq. (2) enforces the steady-state altitude given by h_c. This altitude, however may not be the optimal steady-state altitude. Therefore, a new parameterization is used that allows an exponential damping of the sinusoid trajectory along with a new exponential term on the steady-state altitude variable. This parameterization takes the form

$$h = \exp[-\eta_1(r/r_f)][h_a \cos(2\pi/r_f)r + h_b \cos(4\pi/r_f)r] + \exp[-\eta_2(r/r_f)]h_c + h_d \tag{3}$$

Using this new altitude parameterization, the damped periodic trajectories are optimized like method 1, maintaining continuity of the states, with the additional damping term η_2 and harmonic coefficient h_d terms added as design variables for the first period. Pure periodic and steady-state trajectories are exactly the same as method 1.

Method 3

Alluding back to the argument that damped periodic trajectories damp out to steady-state trajectories that may not be optimal, another strategy is used. Instead of using a new altitude parameterization, the constraint that the initial kinetic energy (KE) to potential energy (PE) ratio for the damped periodic trajectories be the same as the resulting optimal steady-state ratio from method 1. This ratio has a value of KE/PE = 0.1888. Aside from this additional constraint, the optimization of damped periodic trajectories follows the same methodology as method 1. Pure periodic and steady-state trajectories are also the same as method 1.

Vehicle Dynamic Model

The equations of motion used are for flight in a vertical plane over a nonrotating spherical Earth with range as an independent variable. The nonlinear equations of motion are summarized in Chuang and Morimoto[4] and von Eggers Rudd et al.[5] The vehicle model used is the same as that studied by Chuang and Morimoto,[4] which is a modified version HL-20 spaceplane. The aerodynamic and engine models use curve fitted data taken from the available spaceplane literature.[4]

Optimization

Cost Function

The function that is minimized is the range averaged fuel consumption rate. Mathematically, this equation is expressed as[4]

$$J = \frac{1}{r_f}\int_0^{r_f} \frac{T}{g I_{\mathrm{sp}} Ma \cos\gamma}\left(1 + \frac{h}{R_o}\right) \mathrm{d}r \tag{4}$$

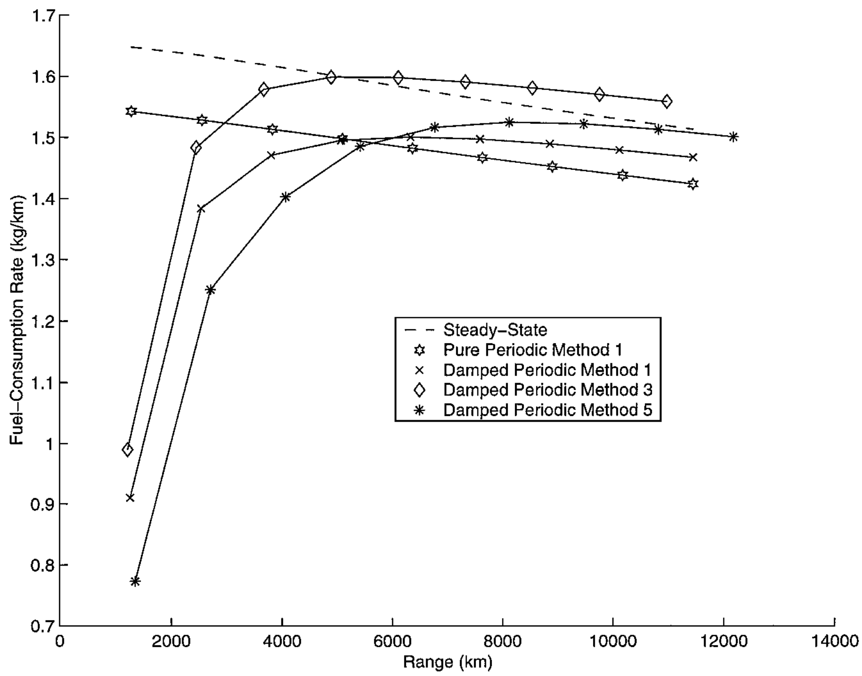

Fig. 1 Range-average fuel consumption vs range.

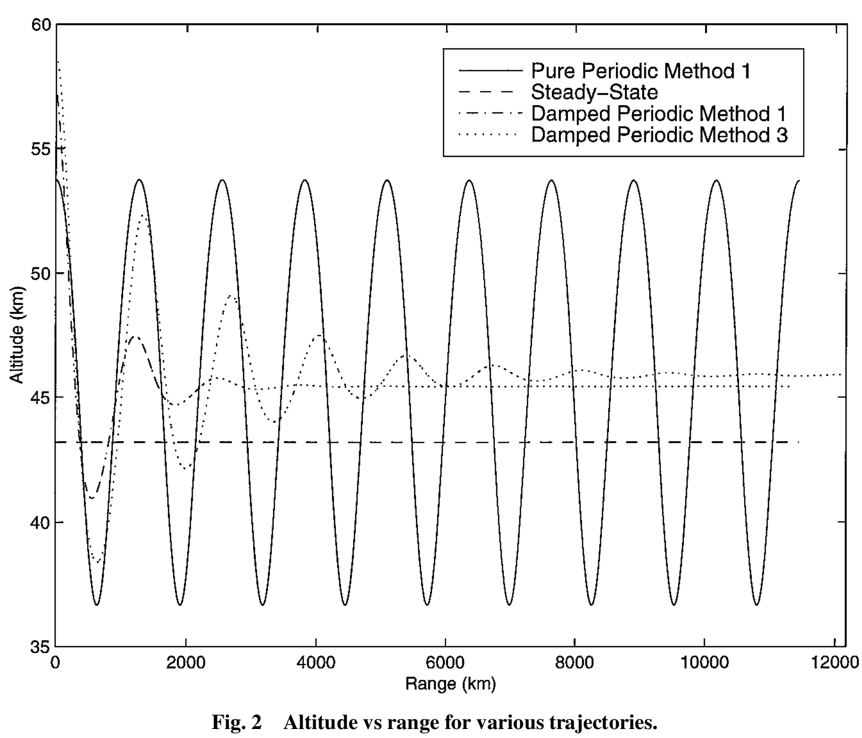

Fig. 2 Altitude vs range for various trajectories.

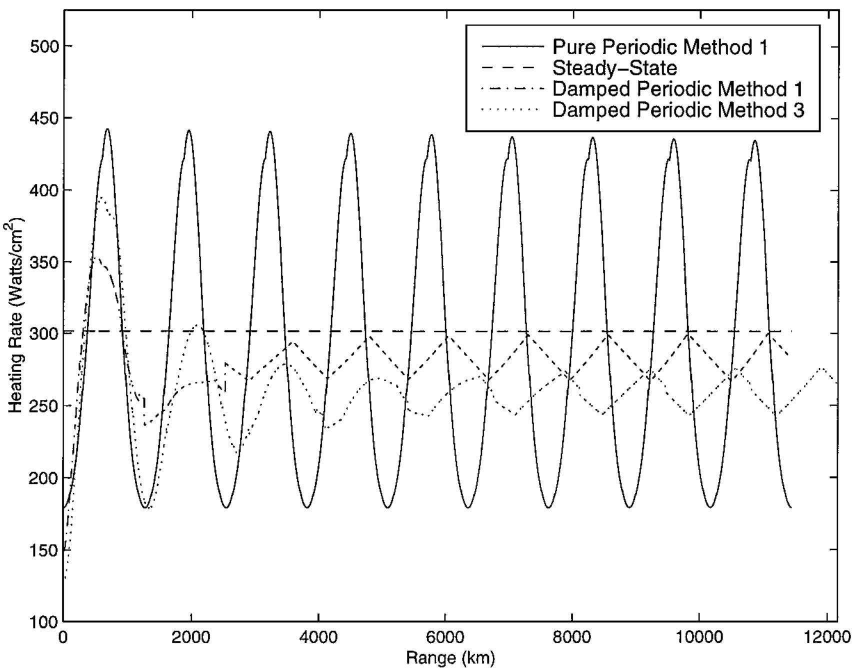

Fig. 3 Heating rate vs range for various trajectories.

where R_o is the mean radius of the Earth. This cost function is used for all cases except for the steady-state constant mass problem. For this case, an instantaneous fuel consumption rate is used:

$$J = (T/gI_{\rm sp}Ma)[1 + (h/R_o)] \tag{5}$$

Optimizer

The parameter optimization is done with the Design Optimization Tools (DOT) software package.[7] A detailed explanation of this optimizer can be found in the DOT users manual.[7]

Optimization Results

Nine periods were optimized for the steady-state and damped periodic hypersonic cruise trajectories. This corresponded to a total range of 11,374 km. Similarly, the purely periodic hypersonic cruise trajectories were also optimized for nine periods corresponding to a range of 11,440 km. Figure 1 displays a plot of the range-averaged fuel-consumption savings vs range for all three methods considered in this study. Notice that damped periodic hypersonic cruise trajectories are more efficient up to period four. Beyond this point, purely periodic cruise trajectories are more efficient.

The poor performance of the damped periodic cruise trajectories beyond period four can be explained by examining the choice of parameterizations and constraints used in the optimization. Figure 2 displays a plot of altitude vs range for steady-state cruise, purely periodic cruise, and damped periodic cruise for two different parameterizations (methods 1 and 3). Notice that the optimal mean steady-state altitude constant h_c is inconsistent with the mean altitude of the damped periodic cruise trajectories. This implies that when the damped periodic cruise trajectories reach steady-state conditions, more energy is required to maintain this higher mean altitude. Method 3 attempts to address this issue by insuring that the damped periodic solutions asymptotically approach the steady-state cruise mean altitude by the ninth period. Indeed, the multiple-period result for this parameterization follows a smooth curve toward the steady-state cruise fuel consumption. A better approach might be to choose a value of the fuel-consumption savings desired at the end of a long-range flight. This may force the damped periodic solutions to adjust to meet these final end conditions.

Figure 3 displays heating rates, respectively, for the steady-state, purely periodic, and damped periodic trajectories. Notice that the suboptimal damped periodic solutions lead to lower heating rates. These properties are important for actual vehicle performance and thermal protection system sizing.

Summary and Conclusions

Previous research has demonstrated that damped periodic hypersonic cruise trajectories have superior fuel-consumption savings over steady-state cruise and periodic trajectories for a single period. This work has investigated whether damped periodic hypersonic cruise trajectories exhibit the same performance over multiple periods required for long-range flight. Preliminary results suggest that for the altitude parameterizations considered in this study, damped periodic hypersonic cruise trajectories exhibit better fuel-consumption savings over the first few periods of a long-range trajectory. However, beyond the fourth period of a long-range hypersonic flight, purely periodic solutions achieve better fuel-consumption savings. Optimization results also suggest that there is an optimal tradeoff between energy dissipation through a nonisothermal atmosphere and fuel-consumption savings for multiple periods. A better understanding of the vehicle/atmospheric interactions will possibily enhance fuel-consumption savings for damped periodic hypersonic trajectories. The major disadvantage of this work is that the optimization is conducted in a piecewise manner. Therefore, a full TPBVP optimization will have to be completed to determine whether periodic, damped periodic, or a combination of the two achieves better long-range fuel-consumption savings.

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